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Valerii Vasil'evich Kozlov (On his Sixtieth Birthday)[☆]

On the 1 January 2010, Valerii Vasil'evich Kozlov, the remarkable mathematician and mechanical engineer, vice-president of the Russian Academy of Sciences and director of the V.A. Steklov Mathematical Institute, celebrates his sixtieth birthday.

He entered the Mechanics Branch of the Faculty of Mechanics and Mathematics of the M.V. Lomonosov Moscow State University in 1967, and studied in the Department of Theoretical Mechanics. He trained as a scientist in the heyday of the Moscow Mathematics School, influenced by the lectures and work of outstanding mechanical engineers and mathematicians such as L.I. Sedov, A.N. Kolmogorov, V.M. Alekseyev, and V.I. Arnold. In his first scientific course work he demonstrated ability to work independently, and by the fifth course he had grown into a mature researcher capable of grasping problems which arose at the end of the previous century, and of tackling them. In the autumn of 1971, the Russian translation of the first volume of Poincaré's *New Methods of Celestial Mechanics* was published. From this time

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on, among the main reference points in his scientific activity were Poincaré's ideas, which at the end of the twentieth century were still not entirely understood and still underestimated. He was one of those who knew how to explain to the scientific community the depth and strength of Poincarés research, and he creatively developed and improved upon many of Poincarés investigations. He not only solved problems that were formulated a century before but also opened up new areas of science.

Kozlov began his scientific work by focusing on rigid body dynamics, and he later returned to this subject repeatedly. Working on the Goryachev–Chaplygin problem, he obtained an extension of the Louiville–Arnold theorem on phase toruses to the case where some integrals are not general but particular. He then demonstrated the absence of new cases of integrability in the problem of the motion of an asymmetrical heavy rigid body with a stationary point, which was the solution of a problem posed by Poincaré at the end of the nineteenth century. In the course of this research, Kozlov developed a number of effective Poincaré methods that made it possible to establish the non-integrability of a given Hamiltonian system. Most of these methods require the presence in the system of a small parameter, the system being integrable when this parameter has a zero value. The improved versions of Poincaré methods that he proposed proved to be applicable to a considerably broader class of systems. In 1974, he defended his Candidate of Science dissertation ahead of schedule, and was appointed by D.Ye. Okhotsimskii to work as a junior member of the research staff in the Department of Theoretical Mechanics. In 1978, Kozlov defended his doctoral dissertation, the results of which became the basis of his book *Methods of Qualitative Analysis in Rigid Body Dynamics*. The reader could easily see that the methods indicated went far beyond the methods used at that time by rigid body dynamics specialists. Besides the traditional methods, they included ideas from dynamics, the analytical theory of differential equations, variational analysis, and number theory. This breadth of views and approaches became a distinguishing feature of Kozlov's entire creative activity.

In 1979, he discovered qualitatively new obstacles to integrability, not reflected in the classical literature, based on an analysis of the topology of configuration space. The requirements of the Hamiltonian in such a situation are extremely simple and natural. This work began a new chapter in Hamiltonian mechanics – the theory of topological obstacles to integrability.

At the beginning of the 1980 s, the focus of his scientific interests shifted to stability theory, where he managed to develop Lyapunov's first method significantly. This work was based on refined notions from functional analysis relating to the existence of solutions of differential equations when power series (or more complex series) for these solutions diverge in time. Some of the required theorems of existence were obtained independently by him, and some were proved, on his request, by specialists working on the theory of functions and consequently became standard tools in the analysis of the stability of equilibrium positions. One of the most spectacular achievements of this activity was the complete proof of Earnshaw's famous theorem on equilibrium instability in a force field with a harmonic potential. These problems were the subject of the monograph *Asymptotics of the Solutions of Non-linear Systems of Differential Equations*, written jointly with S.D. Furta.

At the same time, he solved the following natural question: what equations will be obtained if the variational problem is examined for a mechanical system with non-integrable constraints? The differential equations giving the solution of this problem proved not to be the same as the equations of non-holonomic mechanics. It was soon ascertained that systems exist for which the equations obtained in this way have considerably greater goodness-of-fit than non-holonomic equations. These are systems for which natural models have a variational nature. The systems obtained were called "vaconomic". Vaconomic mechanics is well known to specialists. Such systems are being investigated both in Russia and abroad.

Beginning with his doctoral dissertation and up to the middle of the 1980 s, he wrote several works on the application of variational principles in the dynamics of natural Lagrangian systems primarily to the problem of the existence of solutions emerging on the boundary of the region of possible motion, in particular special periodic solutions – librations, and to problems of stability theory. The results obtained were summarised in a review article in *Uspekhi Matematicheskikh Nauk* that had wide resonance among specialists.

In 1985, the third volume of the series Advances in *Science and Engineering. Modern Problems of Mathematics. Fundamental Directions* was published. It was called *Mathematical Aspects of Classical and Celestial Mechanics*. Its authors were V.I. Arnold, V.V. Kozlov, and A.I. Neishtadt. This oustanding book ran to several editions, including an English language edition. It continues to serve as a universal reference book and as a source of references on problems of classical mechanics. In 1986, Kozlov was a guest speaker at the International Congress of Mathematicians in Berkeley. His paper addressed various aspects of the phenomenon of non-integrability in Hamiltonian systems and aroused great interest; a review was published in *Uspekhi Matematicheskikh Nauk*.

At the end of the 1980 s he became interested in the problem of the realization of constraints. It is well known that constraints in mechanics are only a certain abstraction giving convenient and effective mechanical models. The physical and mechanical effects behind this abstraction began to interest specialists at the turn of the nineteenth and twentieth centuries. Work by Lecornu, Klein and Prandtl set a course towards realising bilateral constraints. Kozlov developed similar ideas for the case of unilateral constraints. The theory of unilateral constraints directly touches on impact theory, and these results were naturally included in the monograph, written jointly with D.V.Treshchev, *Billiards. A Genetic Introduction to the Dynamics of Systems with Impacts*.

At the beginning of the 1990s, Kozlov took a general look at invariants of dynamical systems, introducing so-called tensor invariants which contained, as special cases, first integrals, invariant measures and other integral invariants, fields of symmetry, etc. It is well known that each of these invariants carries non-trivial information on dynamics, but previously they were considered individually. He proposed general methods for finding impediments to the existence of such invariants in specific systems, which was a development of his early work on non-integrability. These results comprised one of the chapters of his book *Symmetries, Topology and Resonances in Hamiltonian Mechanics*, which was soon to appear.

It should be pointed out that the results he obtained concerning the various aspects of the problem of non-integrability cannot in any instance be regarded as exclusively negatory. Thus, the non-existence of an analytical integral is closely related, conversely, to the existence in the perturbed system of a large number of non-degenerate periodic solutions. The phenomenon of integrability and even the fact of the existence of some non-trivial (*a priori* non-obvious) tensor invariant are very untypical from the dynamics viewpoint. Systems possessing such "additional" symmetry are rare. Their study merits special attention because, in a number of cases, it has been established that they are of fundamental importance in physics. However, if in a certain class it is possible to analyse in sufficient detail the problem of the existence of an additional invariant, then those few systems for which the absence of an invariant cannot be proved can lay claim to a "special role" in the given class.

It is well known that, from the viewpoint of the theory of dynamical systems, the main axioms and hypotheses of non-equilibrium statistical mechanics are not very obvious and, formally speaking, incorrect. Their true dynamical significance became the subject of his research from the end of the 1990s onwards. In this area, in particular, he developed the theory of weak limits of solutions to the Louiville's equations, clarifying the paradoxes of statistical mechanics and giving a correct dynamical interpretation of its principles. The results of this work are set out in the book *Gibbs and Poincaré Thermal Equilibrium*.

Of his publication that have appeared in recent time, separate mention must be made of a number of papers in which he discusses the possibilities of transferring ideas of classical Hamiltonian dynamics and statistical mechanics to the quantum case. Here, some classical constructs have clear quantum (non-commutative) analogues, whereas the correct quantum role of others remains very unclear. Nevertheless, the formulation of these problems is of undoubted interest in itself. Directly related to this work are his investigations into the application of ideas of ergodic theory to the study of Vlasov's kinetic equation. We should also mention a number of papers in which he studied linear (autonomous or periodic in time) systems allowing of quadratic forms that do not increase along the solutions. It proved possible to establish numerous, very non-trivial relations between the spectral properties of such systems and the indices of inertia of these forms.

This brief description of his scientific activity leaves no room for discussing the solution of the classical Panlevé problem of the relation between the branching of solutions in the complex time plane and the existence of single-valued first integrals, the solution of the Chaplygin problem of a falling heavy rigid body in an unbounded volume of ideal fluid, the discovery of the complete controllability of a body in fluid with a rigid shell and varying mass geometry, the classification of fully integrable generalized Toda chains, the development of the vortex method for the precise integration of canonical equations, the discovery of non-trivial solutions of the Klein–Gordon equations in de Sitter space with finite action and other interesting results.

He devoted much time and energy to the Faculty of Mechanics and Mathematics of Moscow State University, where he is head of the Department of Differential Equations. He set up the School for Dynamical Systems of Classical Mechanics, which is well known both here and abroad. His students include a corresponding member of the Russian Academy of Sciences, seven doctors of science and 29 candidates of science. His seminars (led jointly with S.V. Bolotin and D.V. Treshchev) are well known, and their subject field goes far beyond the offical remit (problems of classical dynamics and statistical mechanics).

His personal qualities are such that he has often been called upon to take on administrative work, counting on his scientific authority, his ability to see through to the heart of the matter, his loyalty to his colleagues and his fairness and tact. In the 1980s he worked successfully as the vice-dean of the Faculty of Mechanics and Mathematics of Moscow State University, and then, for 10 years, was pro-rector of Moscow State University; in the period from 1998 to 2001 he was deputy minister and the principal scientific secretary of the Russian Higher Certification Commission (VAK). Since 2001 he has been a vice-president of the Russian Academy of Sciences, and since 2004 the director of the V.A. Steklov Mathematical Institute of the Russian Academy of Sciences. He is also a member of the Russian President's Council for Science, Education and Technologies. He somehow always manages to continue and expand his research and teaching, in spite of his organizational work and the drain on his energy and time that this involves.

He is editor-in-chief of the journals *Izvestiya Ross. Akad. Nauk Seriya Matematicheskaya* and *Regular and Chaotic Dynamics*. He is a long-established author and reviewer and is now a member of the editorial board of the journal *Prikladnaya Matematika i Mekhanika.*

His scientific services have received widespread recognition. He has been awarded the Lenin Komsomol Prize, the First-Grade Lomonosov Prize, the S.A. Chaplygin Prize of the USSR Academy of Sciences, the State Prize of the Russian Federation, the S.V. Kovalevskaya Prize of the Russian Academy of Sciences, the Henri Poincaré Gold Medal of the International Federation of Nonlinear Analysts (IFNA), and also the Leonhard Euler Gold Medal of the Russian Academy of Sciences. In 1997 he was made a corresponding member and in 2000 a full member of the Russian Academy of Sciences. In 2003 he was made a foreign member of the Serbian Scientific Society. He was awarded an Order of Honour and third- and fourth-class orders "For Services to the Homeland".

The editorial board, editorial staff and readers of the journal, and his colleagues and students send him warm birthday greetings and wish him robust health, new scientific discoveries, success in his organizational activity, and happiness.

LIST OF V.V. KOZLOV'S PRINCIPAL SCIENTIFIC PUBLICATIONS

1973

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1974

The non-existence of analytical integrals of canonical systems similar to integrable systems. *Vestnik. MGU. Ser. 1. Matematika, Mekhanika*, 1974, (5), 74–79.

1975

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1976

The non-existence of analytical integrals close to the equilibria of Hamiltonian systems. *Vestnik. MGU. Ser. 1. Matematika, Mekhanika*, 1976, (1), 110–115.

On a Poincaré problem. *Prikl. Mat. Mekh.*, 1976, **40**(2), 352–355.

The principle of least action and periodic solutions in problems of classical mechanics. *Prikl. Mat. Mekh.*, 1976, **40**(3), 399–407.

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1977

A qualitative analysis of the motion of a rigid body in the Goryachev–Chaplygin case. *Prikl. Mat. Mekh.*, 1977, **41**(2), 225–233.

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Libration in systems with many degrees of freedom. *Prikl. Mat. Mekh.*, 1978, **42**(2), 245–250 (coauthor S.V. Bolotin).

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1980

Periodic oscillations of a composite pendulum. *Prikl. Mat. Mekh.*, 1980, **44**(2), 238–244.

The asymptotic solutions of the equations of dynamics *Vestnik. MGU. Ser. 1. Matematika, Mekhanika*, 1980, (4), 84–89 (coauthor S.V. Bolotin).

The oscillations of one-dimensional systems with a periodic potential. *Vestnik. MGU. Ser. 1. Matematika, Mekhanika*, 1980, (6), 104–107.

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1981

Instability of equilibrium in a potential field. *Uspekhi Mat. Nauk*, 1981, **36**(1), 209–210.

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